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ON MATHEMATICAL PROJECTION OF NIGERIA POPULATION USING NUMERICAL TECHNIQUES

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ABSTRACT

Population projection has been a good alternative to real population census in under-developing country like Nigeria. This study project twenty-five (25) years of Nigeria population between 2006 and 2031. We used Malthus model equation for the projection population using 2006 conducted Nigeria census as initial population. We employed two numerical techniques: Exponentially collocation approximation method and Differential Transformation method to solve Malthus model for the year under consideration. The projected Nigeria population for twenty-five (25) years obtained are in complete agreement with the analytical solution of Malthus model.

Keywords: *Malthus model equation, population projection, twenty-five years, Exponentially collocation approximation method (EFCAM), Differential transformation method (DTM), analytical solution.*

INTRODUCTION

Thomas Malthus (1766 – 1834), a mathematician, was the first to give serious thought to exponential growth in population. He propose that unless the population is check in some ways, it double their numbers in every quarter of a century. Nigeria population census can be traceable back to 19th and 20th centuries, the high population growth rate showed its statistical revelation in the census of 1952/53 with total population of 30.4 million with a growth rate of 2.1%. The second population census conducted in 1963 put Nigeria`s population at 55.6 million and growth rate at 5.6% per annum. Though, the 1973 census was controversial and the result generally unacceptable, the 1991 census put Nigeria`s population and its growth rate at 88,990,220 million and 2.6% respectively (NPC). The last population census conducted in 2006 revealed that Nigeria`s population is 140,431,790 million, while 69,086,302 females and 71,345,488 males, which means that the Nigerian population has increased from 88,992,220 to 140,431,790 in just 15 years. Today, Nigeria is ranked the most populated country in Africa and 7th in the world (UN 2010).The work of M.S Ewugi and Illiyasu Yakubu (2012) state that this trend indicates that Nigeria is one of the fastest growing countries in the world. Her population therefore, is expected to double in less than 25 years with population growth rate of 2 -3.3% (UN, 2001). This high population growth rate is essentially due to persistent high fertility rate of 5.3 children per woman and decreasing death rate from 27 to 15 per 1000 persons (UN, 2010).

Therefore, the aim of this study is to employ two numeric-analytic techniques: Exponentially Fitted Collocation Approximation Method (EFCAM) and Differential Transformation Method (DTM) for the numerical solution of the Malthusian population model and project 25 years Nigeria population.

MATERIAL AND METHODS

FORMULATION OF MALTHUS POPULATION MODEL

Let "P" be the size of the population of human being and "t" be the independent variable.

We assume that both birth and death are proportional to the population size and the time interval. Thus, we have.

Birth α P and dt (1)

Birth= β Pdt (2)

Where β is a constant, is the change in time and P is the size of population.

Similarly,

Death αP and dt (3)

Death= ΩPdt (4)

We denote the increase of total population by Pdt in the time interval t.

From equations (2) and (4), we obtained

$dP=\beta Pdt-\Omega Pdt$ (5)

$dP=P(\beta-\Omega)dt$ (6)

$dP=\lambda Pdt$ (7)

Here $\lambda= (\beta- \Omega)$

$dP/dt=\lambda P$ (8)

After separating the variable to get P, we can now solve the first order differential equation.

$P(t)=P_o e^{\lambda t}$ (9)

Where P_o=Initial population at a given time t.

λ = population growth rate

t = the time the population grows.

P(t) = what the population grows to after time t

We have the following assumptions:

If $\lambda>0$ we have an exponential growth

If $\lambda<0$ we have an exponential decay

If $\lambda=0$ we have constant growth (no change in population size).

Equation (9) is called Malthus population model.

FORMULATION OF TECHNIQUES

Exponentially Fitted Collocation Approximate Method (EFCAM).

In this section, we consider the approximate solution of the form:

$$P_N(t) \approx \sum_{q=0}^{N} s_q t^q \quad ....(10)$$

The Exponentially fitted approximate solution propose by Falade K.I (2015) of the form:

$$P_N(t) \approx \sum_{q=0}^{N} s_q t^q + \tau_1 e^t \quad ....(11)$$

Where t = time, N = Computational length and τ_1 = free tau parameter

In order to solve equation (9), we obtained first derivative of equation (10) and substitute into (9), we obtained

$$\sum_{q=0}^{N} q s_q t^{q-1} = \lambda \sum_{q=0}^{N} s_q t^q \quad ....(12)$$

The expansion of equation (12) and taking computational length N= 8, we obtained

$$\begin{cases}(s_1+2s_2t^1+3s_3t^2+\cdots+8s_8t^7)- \\ \lambda[s_0+s_1t^1+s_2t^2+\cdots+s_8t^8]=0\end{cases} \quad ......(13)$$

Slightly perturbed and collocate equation (13), we obtain

$$\begin{cases} -\lambda s_0+[1-\lambda t_i]s_1+ \\ [2t-\lambda t_i^{\,2}]s_2+[3t_i^{\,2}-\lambda t_i^{\,3}]s_3+ \\ [4t_i^{\,3}-\lambda t_i^{\,4}]s_4+[5t_i^{\,4}-\lambda t_i^{\,5}]s_5+ \\ [6t_i^{\,5}-\lambda t_i^{\,6}]s_6+[7t_i^{\,6}-\lambda t_i^{\,7}]s_7+ \\ [8t_i^{\,7}-\lambda t_i^{\,8}]s_8-\tau_1T_8(t_i)=0 \end{cases} \quad ....(14)$$

Where $t_i=\frac{i}{10}; i=1,2,\dots,9$

Taking P_o as the last Nigeria census 2006 and consider approximate solution (11), we have

$s_0+\tau_1 e^0=P_o$ ....(15)

Equations (14) and (15) altogether, we obtained ten (10) algebraic linear equations in 10 with unknown constants. Thus, we put the 10 algebraic equations in Matrix form as

AS=G ....(16)

Where

$$A=\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14}\dots\dots & p_{19} & -T_8(t_1) \\ p_{21} & p_{22} & p_{23} & p_{24}\cdots\cdots & p_{29} & -T_8(t_2) \\ p_{31} & p_{32} & p_{33} & p_{34}\cdots\cdots & p_{39} & -T_8(t_3) \\ & & & \vdots & & \\ & & & \vdots & & \\ p_{91} & p_{92} & p_{93} & p_{94}\cdots\cdots & p_{99} & -T_8(t_9) \\ 1 & 0 & 0 & 0 & & 1 \end{pmatrix} \quad S=\begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_8 \\ \tau_1 \end{pmatrix} \quad G=\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ P_0 \end{pmatrix}$$

Here,

$$\begin{cases} P_{11}=-\lambda & when\ q=0 \\ P_{12}=[1-\lambda t_1] & when\ q=1 \\ P_{13}=[2t-\lambda t_1^{\,2}] & when\ q=2 \\ P_{14}=[3t_i^{\,2}-\lambda t_i^{\,3}] & when\ q=3 \\ \vdots & \\ \vdots & \\ \vdots & \\ P_{19}=[8t_i^{\,7}-\lambda t_1^{\,8}] & when\ q=8 \end{cases}$$

The ten (10) unknown constants $s_0, s_1, s_2, s_3, \dots, s_8, \tau_1$ are obtain using MAPLE 18 software and substitute into Exponentially fitted collocation approximate method (11).

Differential Transformation Method (DTM).

The concept of Differential Transformation Method was first proposed by Zhou (1986) and it was applied to solve linear and non-linear initial value problems in electric circuit analysis. The Differential Transformation Method (DTM) is very effective and powerful for solving various kind of differential equations. The work of C.L. Chen and Y.C. Liu (1998) employed differential transformation method to solve two-point boundary value problems, Fatma Ayaz (2004), applied differential transform method to differential-algebraic equations and Figen Kangalgil and Fatma Ayaz (2009) Solitary wave solutions for the KdV and mKdV equations by differential transform method and just to mention a few.

Suppose the differential transformation of the function p(t) is defined as follows:

$$P(k)=\frac{1}{k!}\left[\frac{d^k p(t)}{dt^k}\right]_{t=0} \quad ....(17)$$

Where p(t)the original is function and P(k) is the transformation function.

Here $d^k/(dt^k)$ means that k the derivate with respect to t. The differential inverse transform of P(k) is defined as

$$p(t)=\sum_{k=0}^{\infty}P(k)t^k \quad ....(18)$$

Combining equations (17) and (18), we obtained

$$P(k)=\frac{1}{k!}\left[\frac{d^k}{dt^k}\sum_{k=0}^{\infty}P(k)t^k\right]_{t=0} \quad ....(19)$$

Equation (19) is called approximate solution of the population functions .

Base on the above definitions, the fundamental mathematical operation performed by the differential transform method is show in table 1

Table 1: ONE DIMENSIONAL DIFFERENTIAL TRANSFORMATION

FUNCTIONAL FORM	TRANSFORMED FORM
P(t) = w(t) v(t)	P(k) = W(k) V(k)
p(t) = ηv(t)	P(k) = ηV(k) , is a constant
p(t) = $\frac{d^m p(t)}{dt^m}$	P(k) = $\frac{(k+m)!}{k!}P(k+m)$
p(t) = ℓ^t	P(k) = $\frac{1}{k!}$
p(t) = ℓ^t	P(k) = $\frac{\lambda^k}{k!}$
p(t) =sin $(ct+\beta)$	P(k) = $\frac{C^k}{k!}\sin\left(\frac{\pi k}{2}+\beta\right)$
p(t) = cos $(ct+\beta)$	P(k) = $\frac{C^k}{k!}\cos\left(\frac{\pi k}{2}+\beta\right)$
p(t) = t^n	P(k) = $\delta(k-n)$, is constant delta $\begin{cases}1, k=m \\ 0, k\neq m\end{cases}$

IMPLEMENTATION OF THE NUMERICAL TECHNIQUES

EFCAM Implementation

Consider the equations (14), (15) and (16) with the computation length 8,

Morever, P_o =140,431,790 million (2006 Nigeria population census) and λ = Nigeria population growth rate = 2.67 % Source: United Nations Department of Economic and Social Affairs: Population Division (2019)

We have matrix:

$$A = \begin{pmatrix} -0.0267 & 0.9973 & 0.1997 & 0.0299 & 0.0039 & 0.00049 & 0.000059 & 0.0000069 & 0.00000079 & -0.4219 \\ -0.0267 & 0.9946 & 0.3989 & 0.1197 & 0.0319 & 0.00790 & 0.00191 & 0.000440 & 0.0001023 & -0.4219 \\ -0.0267 & 0.9919 & 0.5975 & 0.2692 & 0.1077 & 0.04043 & 0.01456 & 0.005097 & 0.0017478 & 0.9886 \\ -0.0267 & 0.9893 & 0.7957 & 0.4783 & 0.2553 & 0.12772 & 0.06133 & 0.028620 & 0.0130890 & 0.0400 \\ -0.0267 & 0.9866 & 0.9933 & 0.7467 & 0.4983 & 0.31166 & 0.18708 & 0.109160 & 0.0623950 & -1.0000 \\ -0.0267 & 0.9838 & 1.1904 & 1.0742 & 0.8605 & 0.64592 & 0.46531 & 0.325840 & 0.223500 & 0.04000 \\ -0.0267 & 0.9813 & 1.3869 & 1.4608 & 1.3655 & 1.19601 & 1.00527 & 0.821340 & 0.657295 & 0.9886 \\ -0.0267 & 0.9786 & 1.5829 & 1.9063 & 2.0371 & 2.03925 & 1.95908 & 1.829400 & 1.673240 & -0.4219 \\ -0.0267 & 0.9759 & 1.7784 & 2.4105 & 2.8985 & 3.26473 & 3.52875 & 3.707310 & 3.814880 & -0.4219 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.00000 & 0.000000 & 1.00000 \end{pmatrix}$$

$$S = \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ \tau_1 \end{pmatrix} \qquad G = \begin{pmatrix} 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 0.00000000 \\ 140{,}431{,}790 \end{pmatrix}$$

Using Maple 18 software code:

Substitute the output values into approximation solution (11), we obtain approximate population for equation (9),

$$P(t)_{Nigeria\ projected\ population}(EFCAM) \approx \begin{cases} 140{,}431{,}790 + 3.749528793E10^{6}t \\ 50056.209390000t^{2} + 445.500263500000t^{3} \\ 2.9737142590000t^{4} + 0.0158796341100t^{5} \\ 0.000070664303t^{6} + 2.69631909E10^{-6}t^{7} \\ 8.66741263E10^{-10}t^{8} + 5.11163252E10^{-10}e^{t} \end{cases} \qquad (20)$$

DTM Implementation

Consider equation (9)

$$\frac{dP}{dt} = \lambda P$$

subject to initial population P(0) =140,431,790 (Nigeria 2006 census)

Taking differential transformation of equation (9) by using table 1 and we obtained recurrence relation of the form

$$P(K+1) = \frac{\lambda P(K)}{(K+1)} \qquad \text{.....................(21)}$$

Where P(K) is the differential transformation of p(t) and the transformation of initial population

P(0)=140,431,790 ....(22)

Substitution equations (22) when k = 0,1,2,…,8 into equation (21), we obtain the following

$$
P(k) = \begin{cases} P(0) = 140{,}431{,}790 \\ P(1) = \dfrac{3749528793}{1000} \\ P(2) = \dfrac{1001124187731}{20000000} \\ P(3) = \dfrac{89100052708059}{200000000000} \\ P(4) = \dfrac{23789714073051753}{8000000000000000} \\ P(5) = \dfrac{6351853657504818051}{40000000000000000000} \\ P(6) = \dfrac{565314975517928806539}{8000000000000000000000000} \\ P(7) = \dfrac{150939098463286991345913}{56000000000000000000000000000} \\ P(8) = \dfrac{40300739289697626689358771}{4480000000000000000000000000000000} \end{cases} \qquad k = 0,1,2,\dots,8
$$

Therefore, the closed form of the DTM projected population of equation (9) can be easily written

$P(t)_{Nigeria\ projected\ population}(DTM) \approx$

$$
\begin{cases} 140{,}431{,}790 \\ + \dfrac{3749528793}{1000}t \\ + \dfrac{1001124187731}{20000000}t^2 \\ + \dfrac{89100052708059}{200000000000}t^3 \\ + \dfrac{23789714073051753}{8000000000000000}t^4 \\ + \dfrac{6351853657504818051}{40000000000000000000}t^5 \\ + \dfrac{565314975517928806539}{8000000000000000000000000}t^6 \\ + \dfrac{150939098463286991345913}{56000000000000000000000000000}t^7 \\ + \dfrac{40300739289697626689358771}{4480000000000000000000000000000000}t^8 \end{cases} \qquad (23)
$$

Table 2: Nigeria Projected Population (Analytical, EFCAM and DTM)

Year (t)	2006 Census	2007	2008	2009	2010
Analytical Population	140,431,790	144,231,824	148,134,685	152,143,156	156,260,095
EFCAM Population	140,431,790	144,231,824	148,134,685	152,143,156	156,260,095
DTM Population	140,431,790	144,231,824	148,134,685	152,143,156	156,260,095
$E_{(t\ (FCAM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
$E_{(t\ (DTM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Year	2011	2012	2013	2014	2015
Analytical Population	160,488,436	164,831,195	169,291,468	173,872,434	178,577,359
EFCAM Population	160,488,436	164,831,195	169,291,468	173,872,434	178,577,359
DTM Population	160,488,436	164,831,195	169,291,468	173,872,434	178,577,359
$E_{(t\ (EFCAM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
$E_{(t\ (DTM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
Year	2016	2017	2018	2019	2020
Analytical Population	183,409,598	188,372,595	193,469,889	198,705,115	204,082,003
EFCAM Population	183,409,598	188,372,595	193,469,889	198,705,115	204,082,003
DTM Population	183,409,598	188,372,595	193,469,889	198,705,115	204,082,003
$E_{(t\ (EFCAM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
$E_{(t\ (DTM))}$	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000.
Year	2021	2022	2023	2024	2025
Analytical Population	209,604,389	215,276,208	221,101,504	227,084,431	233,229,254
EFCAM Population	209,604,389	215,276,207	221,101,504	227,084,430	233,229,252
DTM Population	209,604,389	215,276,208	221,101,504	227,084,430	23,322,9253
$E_{(t\ (EFCAM))}$	0.00000000	0.00000001	0.00000000	0.00000001	0.00000002
$E_{(t\ (DTM))}$	0.00000000	0.00000000	0.00000000	0.00000001	0.00000001
Year	2026	2027	2028	2029	2030
Analytical Population	239,540,353	246,022,228	252,679,500	259,516,917	266,539,351
EFCAM Population	239,540,351	246,022,224	252,679,496	259,516,909	266,539,340
DTM Population	239,540,351	246,022,226	252,679,497	259,516,912	266,539,343
$E_{(t\ (EFCAM))}$	0.00000002	0.00000004	0.00000004	0.00000008	0.00000011
$E_{(t\ (DTM))}$	0.00000002	0.00000002	0.00000003	0.00000005	0.00000008
Year	2031				
Analytical Population	273,751,809				
EFCAM Population	273,751,794				
DTM Population	273,751,798				
$E_{(t\ (EFCAM))}$	0.00000015				
$E_{(t\ (DTM))}$	0.00000011				

RESULT DISCUSSION AND CONCLUSION

Result Discussion

Both analytical and numerical population obtained in 2031 (273,751,809 million) projected Nigeria population has shown and establish the Malthusian population theory which say that unless the population check in some way double their number in every quarter of a century. It was observed that by projected population 2031 (273,751,809) Nigeria population has close to double itself according to the Malthusian theory of twenty-five (25) years.

CONCLUSION

This work is aim to assist in estimating Nigeria both inter-census and post-census population. The work is to evolve how the population of Nigeria will be estimated by using Malthus model by doing so, it is hope that reliable and accurate data can obtained for planning and economic activities, which will in a significant way that Nigeria economic development be achieved. It is also providing the public with usual mathematical knowledge so that they can make use of it in their daily endeavor thereby ensuring that they achieved their individual objectives.

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